

CONFERENCE PROCEEDINGS

ICMM 2017

International Conference on
Mathematics and Mechanics

1-2 June 2017, Nice, France

Obtaining the Riemann Zeta Function From its zeros: An Elementary Proof of the Riemann Hypothesis

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Abstract

The author considers in this work, the zeros of the Riemann Zeta Function and from the multiplication of these zeros derived the analytic continuation formula of the Riemann Zeta Function. He eventually obtained the Riemann Zeta Function Through this; He succeeded in obtaining a general formula for the zeros of the analytic continuation formula of the Riemann Zeta Function for all values of complex numbers and showed that these zeros will always be real.

Keywords: *Riemann zeta function; Non-trivial zeros, analytic continuation formula.*

2010 Mathematics Subject Classification: 11H05, 11M06, 11M26.

The Proof of the equation $\zeta\left(\frac{1}{2} + jt\right) = \frac{1}{2}(e^{\pi t} + j \sin j\pi t)$

The Proof of the Riemann Hypothesis

The Proof of the equation $\Gamma(S) = \frac{2^{S-1}\pi^S}{\cos\frac{\pi S}{2}}$

The Proof of the equation $\Gamma(1 - S) = \frac{2^{-S}\pi^{1-S}}{\cos\frac{\pi(1-S)}{2}}$

P versus NP and Birch and Swinnerton-Dyer Conjecture

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ABSTRACT:

In mathematics, the Riemann hypothesis is a conjecture that the Riemann zeta function has its zeros at the negative even integers and

The complex numbers **with** real part (1/2).

It was proposed by Bernhard Riemann (1859). The Riemann hypothesis implies results about the distribution of prime numbers.

Keywords:

Riemann zeta function, Primes, Euler's equation, Taylor series, Complex number, Graphs program.

Extension Classes of Broyden's and Central Broyden's Methods for Solving $\mathcal{F}(x) = 0$

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Abstract

In this paper, we propose two efficient extensions classes based on Broyden's methods and Broyden's central finite difference for solving systems of nonlinear equations; used to increase order convergence of Broyden's method. These simple extensions have more accuracy than the basic Broyden's method. Some numerical examples are given to test the validity of the proposed algorithms and for comparison reasons. Superior results show the efficiency and accuracy of the proposed algorithms and a tremendous improvements in Broydn's methods.

Keywords: Nonlinear systems of equations; Newton's method, Broyden's methods; Quasi Newton method.

Iterative Solutions for Variational Inclusions Problems in Banach Spaces

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Abstract—Variational inclusion problems has be-come the apparatus that is generally used to constrain sundry mathematical equations in other to guarantee the uniqueness and existence of their solutions. The existence of these solutions was earlier studied and proven for uniform Banach Spaces using accretive operators. In this study, we extend the conditions to hold for arbitrary Banach Spaces using uniform accretive operators.

Keywords—Accretive operators, Banach Spaces.

A Bernstein-Szegö Inequality on Hardy Space

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Abstract

The classical Bernstein-Szegö inequality asserts that

$$T'(x)^2 + n^2 T(x)^2 \leq n^2 \max_{y \in \mathbb{R}} |T(y)|^2$$

for each real number x , and for each real trigonometric polynomial T of degree at most n . In this paper, we establish a Bernstein-Szegö inequality on the Hardy space $\mathbb{H}^2(\mathbb{D})$, a vector space of analytic functions on the unit disk \mathbb{D} , by taking bounded linear operators S and T from some reproducing kernel Hilbert space of functions on \mathbb{D} into the Hardy space $\mathbb{H}^2(\mathbb{D})$ and obtain a sharp estimate using Bessel's inequality. The Hardy space is a reproducing kernel Hilbert spaces (RKHS) of functions on \mathbb{D} . One of the interesting properties of a reproducing kernel Hilbert space $\mathbb{H}^2(\mathbb{D})$ of functions on \mathbb{D} is that, for each $x \in \mathbb{D}$ and $f \in \mathbb{H}^2(\mathbb{D})$, there exists $k_x \in \mathbb{H}^2(\mathbb{D})$ satisfying $f(x) = \langle f, k_x \rangle_{\mathbb{H}^2(\mathbb{D})}$.

Keywords: RKHS, Hardy space, Berstein-Szegö Inequality

Boomerang Motion along a Space Bezier Curve

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A displacement can be defined along a space curve. It is important what the space curve is. If we choose a Bezier curve in R^3 with n-control points, then the displacement along this Bezier curve can be controlled. Every new choosing of n-points give us a new Bezier curve and a new displacement invariably the algorithm. In this article we define a displacement along a Bezier curve in R^3 as model boomerang motion and give example using Matlab.

Keywords: Bezier curve, Boomerang, Close space curve

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Wavelet-Based Multi-Resolution Analysis and Compression of 3D Objects

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Abstract— Wavelets have been applied in a wide range of applications, thanks to their particular and exceptional properties that overcame by far other analysis and synthesis techniques. 3D objects can be represented using various techniques going from implicit to explicit mesh and voxels based models. In this paper, we present a new lazy wavelet family and its algebraic designing method. This wavelet family is characterized by two specific filters, namely, analysis and synthesis filters. Both filters are given as the result of an algebraic method by taking into account the constraints for a multi-resolution analysis. The particularity of the proposed design process lies in that it allows the algebraic building of filters of various dimensions and ,thus, with different datasets scopes and ranges. The wavelet can be applied for the multi-resolution analysis of multi-dimensional discrete datasets especially 3D modelled objects using various techniques such as mesh and voxels based models.

Keywords—wavelets; multi-resolution analysis; 3D objects; Compression

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THE GEOMETRY OF BANACH SPACE AND FIXED POINT OF NON-EXPANSIVE MAPPING

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ABSTRACT. Let X be a Banach space, and $B(X)$ and $S(X)$ be the unit ball and unit sphere of X . In this talk, we study the further properties of some known geometric parameters that related to convexity of $B(X)$, and curves on $S(X)$. The relationships of values of these parameters with normal structure, uniformly non-square are obtained. Some existing results about fixed points of non-expansive mapping are improved.

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2010 *Mathematics Subject Classification.* Primary: 46B20, 46C05, and 52A07.

Key words and phrases. Fixed points, Normal structure, ultra-product, Uniformly non-square.

Optimal control system for nonlinear processes with constraints

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An analytical-numerical method for solving a boundary value optimal control problem for dynamical systems described by ordinary differential equations with constrained control and state variables and mixed constraints is proposed. The embedding principle underlying the method is based on the general solution of a Fredholm integral equation of the first kind and its analytic representation; the method permits one to reduce the boundary value optimal control problem with constraints to an optimization problem with free right end of the trajectory. A condition for the existence of a solution has been obtained and methods for an admissible control and optimal solution construction have been developed. The construction of an optimal control is implemented by sequential narrowing a set of admissible controls depending on a value of the functional. The present work is an extension of research outlined in [1–12].

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An Algorithm for 3D Printer Using Delta Robot, Level Surfaces and Matlab

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The main idea of the 3D printer modelling is to determine the border curves of level curves of the level surfaces on a rigid body. If we define the level surface and border curves of level surface of a body, then we can draw the shape of body.

In this article, we study level surfaces and border level curves of level surfaces of a rigid body with Matlab applications.

Keywords: Delta Robot, Level Surfaces, Matlab

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Bubble Dynamics in A Compressible Liquid with Viscous Effects Subject to Ultrasound

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Abstract—A robust computational model, the compressible viscous BIM, has been formulated for modelling microbubble dynamics subjected to high intensity ultrasound. A few techniques have been implemented to the model in order to successful model the surface mode oscillation. Bubble dynamics in an infinity fluid driving by acoustic wave are modelled by using the weakly compressible theory and VCVPF with BIM. The viscous effects are modelled by the local energy conservation, which is the energy to be used by the viscous correction pressure, which is equal to the work produced by the liquid to the gas by the shear stress of the irrotational flow. Our model accord with the Keller-Miksis model for the bubble with initial radius $26 \mu m$ subjected by the ultrasound at pressure amplitude 20 kPa and frequency 130 kHz for eight cycles of oscillations after the wave arriving. The compressible viscous BIM is consistent with experiment results provided by Palachon 2004 with the same parameters when comparing with Keller-Miksis model.

We explored that microbubble dynamics subjected to ultrasound is propagating from the left of the bubble in an infinite fluid. Numerical analyses were carried out for the bubble developed differently surface mode oscillation in various situations. First, the driving pressure of the acoustic wave has been investigated when $p_a=40, 47, 50 kPa$ with the rest of parameters kept same. Secondly, by keeping all the parameters the same except the frequency of the driving acoustic wave $f=85 kHz, f=f_0$ and $f=130 kHz$ to investigate the effects when the driving acoustic wave has the same, below and above frequency with nature frequency of the bubble. Finally, by setting the driving frequency of the ultrasound is equal to the parametric resonance frequencies subjected to the bubble with initial radius $39 \mu m$ to active different volumetric surface mode oscillations.

Proof of the Riemann Hypothesis

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Abstract: The Riemann zeta function is one of the most Euler's important and fascinating functions in mathematics. By analyzing the material of Riemann's conjecture, we divide our analysis in the zeta function and in the proof of the conjecture, which has very important consequences on the distribution of prime numbers.

I. INTRODUCTION

#1. The Riemann Zeta Function:

Let C denote the set of complex numbers. They form a two dimensional real vector space spanned by $1, i$, where i is a fixed square root of -1 , that is,

$$C = \{x + iy : x, y \in \mathbf{R}, i = \sqrt{-1}\}$$

#2. Definition of Zeta function: The Riemann zeta function is the function of a complex variable

$$\zeta(s) := \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots = \sum_{n=1}^{\infty} n^{-s}, s \in C$$

It is conventional to write $s = \sigma + it$ and $\bar{s} = \sigma - it$ where $\sigma, t \in \mathbf{R}$.

As the equality on the second line follows from unique prime factorization, we can say that the equation

$$\zeta(s) := \prod_p \left(1 - \frac{1}{p^s}\right)^{-1}$$

is an analytic statement of unique prime factorization." It is known as the Euler product". This gives us a first example of a connection between the zeta function and the primes. [3]

#3. Definition of complex function, Inverse functions:

Suppose that A, B are two subsets of the complex plane C . Each f tally the sets A and B : $f: A \rightarrow B$ with the restriction that for each $z \in A$ corresponds one and only one point $w = f(z) \in B$, which is a complex function with domain the set A and range the set B . The z is called independent variable and the dependent variable is w . In cases where the above restriction does not apply, i.e. where each value of the variable $z \in A$ matches to more than one value $f(z)$, the f match is called multi-valued function in contrast to the previous case where you use the term single-valued function or simply function. If we set $z = x + iy$ and $w = u + iv$, then the function $w = f(z)$ corresponds to each complex number $z \in A$ with coordinates x and y being two real numbers u and v . In other words, the set A is a set of two real functions $u = u(x, y)$ and $v = v(x, y)$ of two real variables x and y . So a complex function $w = f(z)$ is equivalent to two real functions $u = u(x, y)$ and $v = v(x, y)$ of two real variables.

Example: The complex function: $z^2 = (x + iy)^2 = x^2 - y^2 + 2 \cdot i \cdot x \cdot y$

Is equivalent to the actual functions: $u = x^2 - y^2, v = 2 \cdot x \cdot y$

If a complex function $w = f(z)$ is one to one (1-1) and on (i.e. $z_1 \neq z_2 \Rightarrow f(z_1) \neq f(z_2)$ or $f(z_1) = f(z_2) \Rightarrow z_1 = z_2$ and $f(A) = B$), then the correlation that corresponds to each $w \in V$ where $z \in A$ such that $f(z) = w$, defines a new function, which is called the inverse of the function $f(z)$ and is denoted by f^{-1} , that is $z = f^{-1}(w)$. **So, if we have $Zeta(z_1) = Zeta(z_2) \Rightarrow z_1 = z_2$ in the domain of the set A, if occurring earlier. In this case, $Zeta[z]$ is 1-1, which is acceptable.**

#4. Extensions of holomorphic functions:

One of the main properties of holomorphic functions is uniqueness in the sense that if two holomorphic functions f and g defined in a domain G are equal on a sequence $z_n \in G$,

$\lim_{n \rightarrow \infty} z_n \in G = z_0 \in G$ i.e., $f(z_n) = g(z_n)$ for $n = 1, 2, \dots$, then $f = g$ in G , see Fig.1.

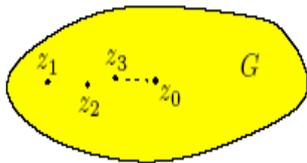


Fig..1

Sequence of points

Of course such a property is not true for functions in real calculus. In particular, if a holomorphic function f is defined in a domain $G_1 \subset C$ and another holomorphic function g is defined in a domain $G_2 \subset C$ with $G_1 \cap G_2 \neq \emptyset$ and $f = g$ on the intersection, then g is determined uniquely by f ; see Fig. 2.

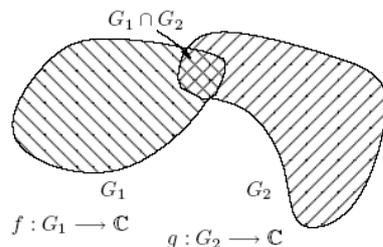


Fig..2

Holomorphic extensions

f cannot be holomorphically extended beyond this disc. A simple example:

$$f(z) = \sum_{n=0}^{\infty} z^n, |z| < 1$$

Obviously the series diverges for values $|z| \leq 1$. However, the function f can be holomorphically extended to the entire complex plane C except $z = 1$, by the formula

$$f(z) = \frac{1}{1-z}$$

A natural question appears: whether the Riemann zeta function can be holomorphically extended beyond the half-plane $\text{Re } s > 1$? The answer is yes, which we show in two steps. The first step is easy, while the second is more difficult.

#5.Extension of $\zeta(s)$ from $\{\text{Re } s > 1\}$ to $\{\text{Re } s > 0\}$

Let us calculate

$$\begin{aligned} (1-2^{1-s}) \cdot \zeta(s) &= (1-2 \frac{1}{2^s}) \cdot (\frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots) = \\ &= (\frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \dots) - 2(\frac{1}{2^s} + \frac{1}{4^s} + \frac{1}{6^s} + \dots) = \\ &= (\frac{1}{1^s} - \frac{1}{2^s} + \frac{1}{3^s} - \frac{1}{4^s} + \frac{1}{5^s} - \frac{1}{6^s} + \dots) = \\ &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^s} \end{aligned}$$

We obtained another formula for $\zeta(s)$,

$$\zeta(s) = \frac{1}{(1-2^{1-s})} \cdot \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n^s}, \text{Re } s > 0, s \neq 1$$

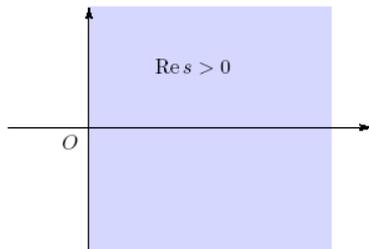


Figure 3

The alternating series converges for $\text{Re } s > 0$

so the alternating series converges in a bigger half-plane (see Fig. 3) than the originally defined function $\zeta(s)$ in (3), but we have to remove $s = 1$ since the denominator

$1 - 2^{1-s}$ vanishes there. This rather easy extension of $\zeta(s)$ from s with $\text{Re } s > 1$ to s with $\text{Re } s > 0$ is already significant as it allows us to formulate the Riemann Hypothesis about the zeros of $\zeta(s)$ in the critical strip.

#6.Functional equations for the Riemann zeta function:

The second step, which provides a holomorphic extension for $\zeta(s)$ from $\{\text{Re } s > 0, s \neq 1\}$ to $\{\text{Re } s < 0\}$, see Fig. 4, was proved by Riemann in 1859. We do not give a proof here of the so-called functional equation, but the proof can be found, e.g. in the book by Titchmarsh. Alternatively one can first holomorphically extend $\zeta(s)$ step by step to half-planes $\{\text{Re } s > k, s \neq 1\}$, where k is any negative integer. For details of this method, you can see the papers [5] and [6].

There are few versions of the functional equation; here we formulate two of them:

$$\text{Eq.set} \begin{cases} \zeta(1-s) = 2 \cdot (2 \cdot \pi)^{-s} \cdot \cos(\pi \cdot s/2) \cdot \Gamma(s) \cdot \zeta(s), \text{Re } s > 0 \\ \zeta(s) = 2 \cdot (2 \cdot \pi)^{s-1} \cdot \sin(\pi \cdot s/2) \cdot \Gamma(1-s) \cdot \zeta(1-s), \text{Re } s < 1 \end{cases}$$

where

$$\Gamma(s) = \int_0^{\infty} x^{s-1} \cdot e^{-x} dx, \text{Re } s > 0$$

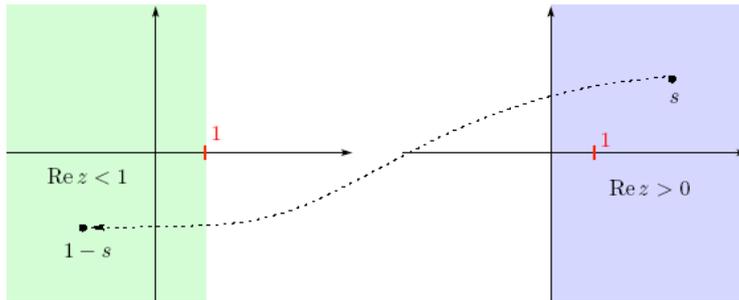


Fig.4

Holomorphic extension domain from the right half-plane to the left half-plane

Before we give more information about the function $\zeta(s)$, we mention that each of the two previous equations, give an extension of $\zeta(s)$ on the entire plane \mathbb{C} except $s = 1$, as it is illustrated in Fig. 4.

The gamma function was already known to Euler. It generalizes the factorial $n!$, namely

$$\Gamma(n) = (n-1)!, n = 0, 1, 2, \dots$$

Its basic properties are that $\zeta(z)$ is holomorphic on the entire plane \mathbb{C} except for the points $z = 0, -1, -2, \dots$. At these points there are simple singularities, called poles, where we have the limits

$$\lim_{z \rightarrow -k} (z+k) \cdot \Gamma(z) = \frac{(-1)^k}{k!}, k = 0, 1, 2, \dots$$

From the definition of the gamma function, it is not clear that it can be extended onto the entire plane, except for $z = 0, -1, -2, \dots$, and that is non-vanishing. Fortunately there are other equivalent definitions of $\zeta(z)$ from which these properties follow more easily; see [8, 9].

Namely, we have:

$$\Gamma(z) = e^{-\gamma \cdot z} \left[z \cdot \prod_{n=1}^{\infty} \left(1 + \frac{z}{n}\right) \cdot e^{-z/n} \right]^{-1}, \Gamma(z) = \lim_{n \rightarrow \infty} \frac{n! \cdot n^z}{z \cdot (z+1) \cdot \dots \cdot (z+n)}$$

From the second formula for the gamma function, we see that $\zeta(z)$ is holomorphic and non-vanishing for $\text{Re } z > 0$.

#7. The Riemann Hypothesis

The Riemann Hypothesis is the most famous open problem in mathematics. Originally formulated by Riemann, while later David Hilbert included the conjecture on his list of the most important problems during the Congress of Mathematicians in 1900, and recently the hypothesis takes place on the list of Clay Institute's seven greatest unsolved problems in mathematics. From the formula of $\zeta(s)$ (as a product) it follows that that the function does not vanish for $\text{Re } s > 1$. Next, by using the functional equation and the fact that $\Gamma(z) \neq 0$ for $\text{Re } z > 0$, we see that $\zeta(s)$ vanishes in the half-plane $\text{Re } s < 0$ only at the points where the function sine is zero, namely we obtain...

$$\zeta(-2k) = 2(2\pi)^{-2k-1} \underbrace{\sin(\pi(-2k)/2)}_{=0} \Gamma(1+2k) \zeta(1+2k) = 0, \quad k = 1, 2, \dots$$

The above considerations do not tell us about the zeros of $\zeta(s)$ in the strip $0 < \text{Re } s < 1$. Actually there are zeros in this strip and they are called nontrivial zeros. Calculation of a number of these nontrivial zeros shows that they are lying exactly on the line $\text{Re } s = 1/2$, called the critical line; see Fig. 5. Now, with the help of computers it is possible to calculate

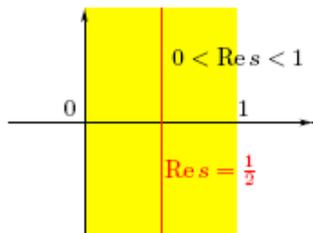


Fig.5

Critical line

an enormous number of zeros, currently at the size of 10^{13} (ten trillion). It is interesting to mention that before the computer era, which began roughly in the middle of the twentieth century, only about a thousand of zeros were calculated. Of course all of these zeros are calculated with high accuracy: they are lying on the critical line. However, there is no proof that really all nontrivial zeros lie on this line and this conjecture is called the Riemann Hypothesis.

Riemann Hypothesis: All nontrivial zeros are on the line $\text{Re } s = 1/2$

Many great mathematicians have contributed to a better understanding of the Riemann Hypothesis. There is no room to even partially list them here. We only mention four of them: Andre Weil (1906 - 1998), Atle Selberg (1917 - 2007), Enrico Bombieri (1940-), and Alain Connes (1947-). The last three received the Fields Medal (in 1950, 1974, and 1982, respectively), which is considered an equivalent to a Nobel Prize in mathematics. The Fields Medal is awarded only to scientists under the age of forty. If someone proves the Riemann Hypothesis and is relatively young, then it certain that he or she will receive this prize.

II. PROOF OF THE RIEMANN HYPOTHESIS

#8. The elementary theorems for the nontrivial zeros of $\zeta(z) = 0$

All previous material, constitute general theoretical knowledge that helps to prove the **Hypothesis**. The essence but the proof is in 3 theorems that will develop and follow specifically in theorem 1, which mostly bordered the upper and lower bound of **the nontrivial zeros roots** in each case of the equations of paragraph 6. Then, theorem 2 is approached by a sufficiently generalized method for finding Lagrange roots, real root of $\zeta(z) = 0$ i.e. special $\text{Re } s = 1/2$. In theorem 3 and using one to one (1-1) imaging of the Zeta Function, calculated on exactly the premise of the Conjecture. So the ability to understand the evidence of the **Riemann's Hypothesis** is given step by step with the simplest theoretical background. Also, we must accept the amazing effort that was achieved by two researchers, namely by Carles F. Pradas [1] also by Kaida Shi [2] who were arrived very close if not exactly with the achievement of a proof.

Lemma 1: (Zeta-Riemann modulus in the critical strip).

- The nonnegative real-valued function $|\zeta(s)| : C \rightarrow R$ is analytic in the critical strip.
- Furthermore, on the critical line, namely when $\Re(s) = 1/2$ one has: $|\zeta(s)| = |\zeta(1-s)|$.
- We have:

$$\lim_{(x,y) \rightarrow (0,0)} |f(s)| |\zeta(1-s)| = 0 \cdot \infty = 1/2 .$$

- $|\zeta(s)|$ is zero in the critical strip, $0 < \Re(s) < 1$, if $|\zeta(1-x-iy)| = 0$, with $0 < x < 1$ and $y \in R$.

Lemma 2. (Criterion to know whether a zero is on the critical line).

Let $s_0 = x_0 + i \cdot y_0$ be a zero of $|\zeta(s)|$ with $0 < \Re(s_0) < 1$. Then s_0 belongs to the critical

line if the condition $\lim_{s \rightarrow s_0} \frac{|\zeta(s)|}{|\zeta(1-s)|} = 1$ is satisfied.

Proof: If s is a zero of $|\zeta(s)|$, the results summarized in Lemma 1. In order to prove whether

s_0 belongs to the critical line, it is enough to look to the limit $\lim_{s \rightarrow s_0} \frac{|\zeta(s)|}{|\zeta(1-s)|} = 1$. In fact

since $|f(s)| = |\zeta(s)| / |\zeta(1-s)|$ and $|f(s)|$ is a positive function in the critical strip,

it follows that when s_0 is a zero of $|\zeta(s)|$, one should have $|\zeta(s_0)| / |\zeta(1-s_0)| = 0/0$ but also

$|\zeta(s_0)| / |\zeta(1-s)| = |f(s_0)|$. In other words, one should have $\lim_{s \rightarrow s_0} \frac{|\zeta(s)|}{|\zeta(1-s)|} = |f(s_0)|$. On the

other hand $|f(s)| = 1$ on the critical line, hence when the condition $\lim_{s \rightarrow s_0} \frac{|\zeta(s)|}{|\zeta(1-s)|} = 1$ is

satisfied, the zero s_0 belongs to the critical line.

Theorem 1:

For the non-trivial zeroes of the Riemann Zeta Function $\zeta(s)$ apply

- i) There exist an upper-lower bound of $\Re s$ of the Riemann Zeta Function $\zeta(s)$ and

$$\text{more specifically in the closed space } \left[\frac{Ln2}{Ln2\pi}, \frac{Ln\pi}{Ln2\pi} \right].$$

- ii) The non-trivial zeroes of the Riemann Zeta function $\zeta(s)$ of upper-lower bound are distributed symmetrically on the straight line $\Re s = 1/2$.

- iii) The average value of upper lower bound of $\Re s = 1/2$.

- iv) If we accept the non-trivial zeroes of the Riemann Zeta Function $\zeta(s)$ as

$$s_k = \sigma_0 + it_k \text{ and } s_{k+1} = \sigma_0 + it_{k+1} \text{ with } |s_{k+1}| > |s_k|, \text{ then}$$

$$t_{k+1} = t_k + \frac{2k\pi}{Ln(n)}, k, n \in N .$$

Proof :

Here, we formulate two of the functional equations from #6.Eq.Set:

$$\zeta(1-s) = 2 \cdot (2 \cdot \pi)^{-s} \cdot \cos(\pi \cdot s/2) \cdot \Gamma(s) \cdot \zeta(s), \text{Re} s > 0$$

$$\zeta(s) = 2 \cdot (2 \cdot \pi)^{s-1} \cdot \sin(\pi \cdot s/2) \cdot \Gamma(1-s) \cdot \zeta(1-s), \text{Re} s < 1$$

We look at each one individually in order to identify and set of values that we want each time.

a). For the first equation and for real values with $\text{Re } s > 0$ and by taking the logarithm of two parts of the equation, we have...

$$\zeta(1-s)/\zeta(s) = 2(2\pi)^{-s} \text{Cos}(\pi s/2)\Gamma(s) \Rightarrow \text{Log}[\zeta(1-s)/\zeta(s)] = \text{Log}[2] - s\text{Log}[2\pi] + \text{Log}[\text{Cos}(\pi s/2)\Gamma(s)] + 2k\pi i$$

but solving for s and if $f(s) = \text{Cos}(\pi s/2)\Gamma(s)$ and from **Lemma 2** if

$$\lim_{s \rightarrow s_0} \zeta(1-s)/\zeta(s) = 1 \text{ or } \text{Log}[\lim_{s \rightarrow s_0} \zeta(1-s)/\zeta(s)] = 0 \text{ we get:}$$

$$s = \frac{\text{Log}[2]}{\text{Log}[2\pi]} + \frac{\text{Log}[f(s)] + 2k\pi i}{\text{Log}[2\pi]} \text{ with } \frac{\text{Log}[f(s)] + 2k\pi i}{\text{Log}[2\pi]} \geq 0$$

Finally, because we need real s we will have $\text{Re } s \geq \frac{\text{Log}[2]}{\text{Log}[2\pi]} = 0.3771$. This is the lower

bound, which gives us the first $\zeta(s)$ of Riemann's Zeta Function.

b). For the second equation, for real values with $\text{Re } s < 1$ and by taking the logarithm of the two parts of the equation, we will have:

$$\zeta(s)/\zeta(1-s) = 2(2\pi)^{s-1} \text{Sin}(\pi s/2)\Gamma(1-s) \Rightarrow \text{Log}[\zeta(s)/\zeta(1-s)] = \text{Log}[2] + (s-1)\text{Log}[2\pi] + \text{Log}[\text{Sin}(\pi s/2)\Gamma(1-s)] + 2k\pi i$$

but solving for s and if $f(s) = \text{Sin}(\pi s/2)\Gamma(1-s)$ and from **Lemma 2**

if $\lim_{s \rightarrow s_0} \zeta(s)/\zeta(1-s) = 1$ or $\text{Log}[\lim_{s \rightarrow s_0} \zeta(s)/\zeta(1-s)] = 0$ we get...

$$s = \frac{\text{Log}[\pi]}{\text{Log}[2\pi]} - \frac{\text{Log}[f(s)] + 2k\pi i}{\text{Log}[2\pi]} \text{ with } \frac{\text{Log}[f(s)] + 2k\pi i}{\text{Log}[2\pi]} \geq 0$$

In the following, because we need real s we will take $\text{Re } s \leq \frac{\text{Log}[\pi]}{\text{Log}[2\pi]} = 0.62286$. This is

the upper bound, which gives us the second of Riemann's Zeta Function of $\zeta(s)$. So, we see that the lower and upper bound exists for $\text{Re } s$ and it is well defined.

ii) Assuming that $s_k = \sigma_{low} + it_k$ and $s'_k = \sigma_{upper} + it_k$ with $\sigma_{low} = \frac{\text{Log}[2]}{\text{Log}[2\pi]}$ and

$\sigma_{upper} = \frac{\text{Log}[\pi]}{\text{Log}[2\pi]}$ we apply $\sigma_1 = \sigma_{low}$ and $\sigma_2 = \sigma_{upper}$. If we evaluate the difference

$$\operatorname{Re} \Delta s'_k = \operatorname{Re} s'_k - 1/2 = \sigma_{upper} - 1/2 = 0.1228 \text{ and}$$

$$\operatorname{Re} \Delta s_k = 1/2 - \operatorname{Re} s_k = 1/2 - \sigma_{low} = 0.1228$$

Then this suggests for our absolute symmetry around the middle value is $\operatorname{Re} s = 1/2$, generally as shown in Figure 6:

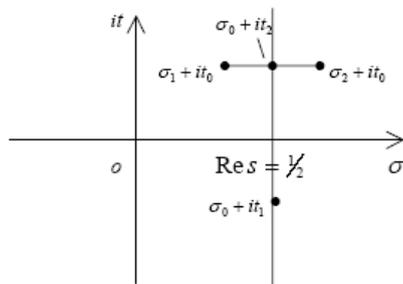


Fig. 6

iii) The average value of the upper lower bound is $\operatorname{Re} s = 1/2$ because

$$\operatorname{Re} s = 1/2 \frac{\operatorname{Log}[2] + \operatorname{Log}[\pi]}{\operatorname{Log}[2\pi]} = 1/2$$

iv. If we accept the non-trivial zeroes of the Riemann Zeta Function $\zeta(s)$ as $s_1 = \sigma_0 + it_1$ and $s_2 = \sigma_0 + it_2$ with $|s_1| > |s_2|$,

and if we suppose that the real coordinate σ_0 of each non-trivial zero of the Riemann Zeta, [2] function $\zeta(s)$ corresponds with two imaginative coordinates t_1 and t_2 , then, we have the following equation group:

$$\begin{cases} \zeta(\sigma_0 + it_1) = \frac{1}{1^{\sigma_0 + it_1}} + \frac{1}{2^{\sigma_0 + it_1}} + \frac{1}{3^{\sigma_0 + it_1}} + \dots + \frac{1}{n^{\sigma_0 + it_1}} + \dots = 0, \\ \zeta(\sigma_0 + it_2) = \frac{1}{1^{\sigma_0 + it_2}} + \frac{1}{2^{\sigma_0 + it_2}} + \frac{1}{3^{\sigma_0 + it_2}} + \dots + \frac{1}{n^{\sigma_0 + it_2}} + \dots = 0. \end{cases}$$

Taking the first equation minus the second, we obtain

$$\begin{aligned} \zeta(\sigma_0 + it_1) - \zeta(\sigma_0 + it_2) &= \sum_{n=1}^{\infty} \left(\frac{1}{n^{\sigma_0 + it_1}} - \frac{1}{n^{\sigma_0 + it_2}} \right) \\ &= \sum_{n=1}^{\infty} \frac{n^{it_2} - n^{it_1}}{n^{\sigma_0 + i(t_1 + t_2)}} = \sum_{n=1}^{\infty} \frac{e^{it_2 \ln n} - e^{it_1 \ln n}}{n^{\sigma_0 + i(t_1 + t_2)}} \\ &= \sum_{n=1}^{\infty} \frac{(\cos(t_2 \ln n) - \cos(t_1 \ln n)) + i(\sin(t_2 \ln n) - \sin(t_1 \ln n))}{n^{\sigma_0 + i(t_1 + t_2)}}, \end{aligned}$$

where

$$n^{\sigma_0+i(t_1+t_2)} = n^{\sigma_0} \cdot n^{i(t_1+t_2)} = n^{\sigma_0} \cdot e^{i(t_1+t_2)\ln(n)} =$$

$$= n^{\sigma_0} \cdot (\cos(t_1 \ln(n)) + i \sin(t_1 \ln(n))) \cdot (\cos(t_2 \ln(n)) + i \sin(t_2 \ln(n))) \neq 0.$$

By enabling the above expression to be equal to zero, we must have

$$\begin{cases} \cos(t_2 \ln n) = \cos(t_1 \ln n), \\ \sin(t_2 \ln n) = \sin(t_1 \ln n) \end{cases} \quad (n = 1, 2, 3, \dots)$$

so, we obtain

$$t_1 = t_2 + \frac{2k\pi}{\ln(n)}, (k = 1, 2, 3, \dots \wedge n = 2, 3, \dots)$$

Thus, theorem 1 has been proved.

Theorem 2:

For **real part of non-trivial** zeroes of the Riemann Zeta function $\zeta(s)$, the upper-lower bound converges in the line $\text{Re } s = 1/2$, by solving the 2 equations of **the Riemann zeta functions**.

Proof:

Using the **Generalized theorem of Lagrange (GRLE)**, [7] to solve an equation, we will attempt to bring the equation in such form, in order to give us the limit of the roots, which converge on one number. Taking advantage of the method, we develop the solution in 2 parts to show the convergence as a number, using the two equations of **the Riemann zeta function**.

a). Solution of the first equation:

For real values with $\text{Re } s > 0$ and if we take the logarithm of the two parts of equation [7] then we have:

$$\zeta(1-s) / \zeta(s) = 2(2\pi)^{-s} \text{Cos}(\pi \cdot s / 2) \Gamma(s) \Rightarrow \text{Log}[\zeta(1-s) / \zeta(s)] = \text{Log}[2] - s \text{Log}[2\pi] +$$

$$\text{Log}[\text{Cos}(\pi \cdot s / 2) \Gamma(s)] + 2k\pi i$$

The resolution analysis will be performed by the **theorem of Lagrange**, which distinguished the best approach for transcendental equations. This will mention some theoretical notes...

Lagrange Expansion: Perhaps the simplest equation upon which to use the foregoing algorithm is $\mathbf{u = a + pg(u)}$ where u is a scalar quantity and g is a scalar function. If $g(u)$ is an analytic function of u in the neighborhood of the point $a = u$ and $|p|$ is sufficiently small, we know by means of straightforward application of the elements of the theory of functions of a complex variable that there will be a unique solution of that is an analytic function of p in the neighborhood of $p = 0$, and that this solution has the form $\mathbf{u = a + p h_1(a) + p^2 h_2(a) + \dots}$ where the coefficients are dependent upon a . It is remarkable that a quite elegant and relatively simple explicit formula exists for the determination of the terms of the foregoing expansion.

Theorem of [Lagrange Expansion]. Let $f(z)$ and $g(z)$ be functions of z analytic on and inside a contour C surrounding $z = a$, and let e satisfy the inequality $|pg(z)| < |z - a|$ for all z

on the perimeter of C. Then the equation in has precisely one root $u = u(a)$ in the interior of C..[7].

Let $f(z)$ be analytic on and inside C. Then

$$f(u) = f(a) + \sum_{n=1}^{\infty} \frac{p^n}{n!} \left(\frac{d}{da} \right)^{n-1} [f'(a)g(a)^n].$$

In this section we bring one **example** to examine our method. Consider the equation $h(x) = x - k - e^{-x} = 0$ with $k > 0$, then $x = k - e^{-x}$. For $k = 2$, using the formula Lagrange with $p = 1$ implies ...

$$x = 2 + \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{d}{dx} \right)^{n-1} (e^{-nx}) \Big|_{x=2} = 2 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} n^{n-1}}{n!} e^{-2n}$$

Choosing the sixteen terms of the above series, gives $x_{16} = 2.1200282339$.

Now by the correlation theory and after using Lemma 2 i.e. we will have:

$\text{Log}[\lim_{s \rightarrow s_0} \zeta(s) / \zeta(1-s)] = 0$ and relations 3 groups fields (but i have interest for one group), and therefore for our case we will get, if we replace $s = x$:

$p_1(x) = y$ which means that $f(y) = p_1^{-1}(y) = x$, but with an initial value

$$x = \frac{2\pi i k + \text{Log}[\pi]}{\text{Log}[2\pi]}$$

and total form from theory Lagrange for the root is..

$$x = \frac{2i\pi k + \text{Log}[\pi]}{\text{Log}[2] + \text{Log}[\pi]} + \sum_{w=1}^q ((-1/\text{Log}[2 * \pi])^w / \text{Gamma}[w + 1]) * (\partial)^{w-1} [(\text{Log}[\text{Sin}[\pi/2 * y]] + \text{Log}[\text{Gamma}[1 - y]])^w]$$

$$\text{with } y \rightarrow \frac{2\pi i k + \text{Log}[\pi]}{\text{Log}[2\pi]}$$

Where q is the count of repetitions of the Sum(Σ). With data $q = 25$, we use a simple program of mathematica to do the proper calculations, and we construct the program:

```
In():
Clear[k, q, y]
k := 1; q := 20; t := (2 i π k + Log[π]) / (Log[2] + Log[π]);
der =
  N[y] +
  Sum[((-1 / Log[2 * π])^w / Gamma[w + 1]) * D[(Log[Sin[π / 2 * y]] + Log[Gamma[1 - y]])^w, {y, w - 1}];
FQ = N[der /. y -> t, 20]
N[Abs[FQ]]
N[1 - 2^(FQ) * (π)^(FQ - 1) * Sin[π * FQ / 2] * Gamma[1 - FQ]]
Plot[1 - 2^(FQ) * (π)^(FQ - 1) * Sin[π * FQ / 2] * Gamma[1 - FQ], {FQ, 0, 10}]
```

```
Out():s:=FQ=0.5000497673504265714 + 3.4362569053495754111i
approximation:= -0.00003021604423530793 - 0.000023485108690195322i
```

Hence the **root s** and we see a good 20 digit approximation, of **upper Re s = 0.5000049** of the order 10^{-5} .

Schematically we illustrate figure 7 **with the program of mathematica:**

Clear [z];

f=Log[1-z]/Log[z]-2*(2*\pi)^{-z}*Cos[\pi*z/2]*Gamma[z];ContourPlot[{Re[f/.z->x+I y]==0,Im[f/.z->x+I y]==0},{x,-15,15},{y,-10,10},ContourStyle->{Red,Blue}]

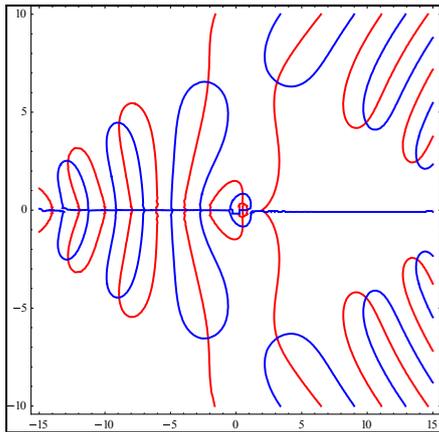


Fig 7

b) Solution of the second equation:

For real values with $\text{Re } s < 1$ and if take logarithm of 2 parts of equation then we have...

$$\zeta(s) / \zeta(1-s) = 2(2\pi)^{s-1} \text{Sin}(\pi s / 2) \Gamma(1-s) \Rightarrow \text{Log}[\zeta(s) / \zeta(1-s)] = \text{Log}[2] + (s-1)\text{Log}[2\pi] + \text{Log}[\text{Sin}(\pi s / 2) \Gamma(1-s)] + 2k\pi i$$

Using the correlation theory and after using Lemma 2 ie.. $\text{Log}[\lim_{s \rightarrow s_0} \zeta(1-s) / \zeta(s)] = 0$ and

relations with 1 from 3 groups fields and therefore for our case we will get, if we replace $s = x$:

$p_1(x) = y$ which means that $f(y) = p_1^{-1}(y) = x$, but with an initial value

$$x = \frac{2\pi i k + \text{Log}[2]}{\text{Log}[2\pi]}$$

and total form from theory Lagrange for the root is..

$$x := \frac{2\pi i k + \text{Log}[2]}{\text{Log}[2] + \text{Log}[\pi]} + \sum_{w=1}^q \left((1 / \text{Log}[2 + \pi])^n / \text{Gamma}[n + 1] \right) * (\partial)_y^{n-1} [(\text{Log}[\text{Sin}[\pi / 2 + y]] + \text{Log}[\text{Gamma}[1 - y]])^n]$$

$$y \rightarrow \frac{2\pi i k + \text{Log}[2]}{\text{Log}[2] + \text{Log}[\pi]}$$

With q being the count of repetitions of the Sum(Σ). Similarly, with data q = 25, we construct a mathematica program in order to perform the proper calculations and we have:

```

In():
Clear[k, q, y]
k := 1; q := 20; t :=  $\frac{2 i \pi k + \text{Log}[2]}{\text{Log}[2] + \text{Log}[\pi]}$ ;
der =
N[y] +
 $\sum_{w=1}^q ((1 / \text{Log}[2 * \pi])^w / \text{Gamma}[w + 1]) * D[(\text{Log}[\text{Cos}[\pi / 2 * y]] + \text{Log}[\text{Gamma}[y]])^w, \{y, w - 1\}];$ 
FQ = N[der /. y -> t, 30]
N[1 - 2 * (2 *  $\pi$ ) ^ (-FQ) * Cos[ $\pi$  * FQ / 2] * Gamma[FQ]]
Plot[1 - 2 * (2 *  $\pi$ ) ^ (-FQ) * Cos[ $\pi$  * FQ / 2] * Gamma[FQ], {FQ, 0, 10}]

```

Out():0.4999502326495734286 + 3.4362569053495754111i
a good approximation with 20 digit ,of lower Re s = 0.49995 of the order 10^{-5} .

Schematically we illustrate figure 8 with the program of mathematica:

```

Clear [z];f=Log[z]/Log[1-z]-2^z*( $\pi$ )^(z-1)*Sin[ $\pi$ *z/2]*Gamma[1-z];ContourPlot[{Re[f/.z->x+I y]==0,
Im[f/.z->x+I y]==0},{x,-15,15},{y,-10,10},ContourStyle->{Red,Blue}]

```

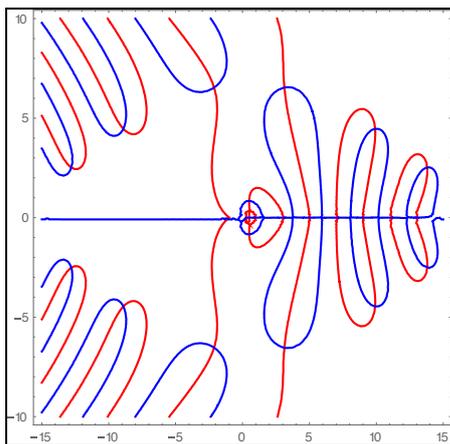


Fig.8

With this analysis, we see that the lower and upper limit converge in the value $\text{Re } s = 1/2$, which are the expected results, proving theorem2.

The equations (Solve(Eq.set[s]), $Zeta[s] = Zeta[1 - s], Zeta[z] = 0$) are for their roots common the real part $\text{Re } s = 1/2$, of the complex roots, as we shall see and in Theorem 3. For this reason the proof of the real and only part of the Riemann $Zeta[s]=0$ us enough proof.

Theorem 3:

The *Riemann Hypothesis* states that *the nontrivial zeros of $\zeta(s)$ have real part equal to 1/2.*

Proof:

Constant Hypothesis $\zeta(s) = 0$.

In this case we use the tho equations of **the Riemann zeta functions**, so if they apply what **they represent the $\zeta(s)$ and $\zeta(1-s)$ to equality.**

Before developing the method, we make the three following assumptions:

I) Analysis of specific parts of transcendental equations, which are detailed...

a). For $z \in \mathbb{C}$

$$\{a^z = 0 \Rightarrow a = 0 \wedge \operatorname{Re} z > 0\}$$

which refers to the inherent function similar to two **Riemann zeta functions** as $(2\pi)^{-z} = 0$ or $(2\pi)^{z-1} = 0$ and it seems that they do not have roots in C-Z because $(2\pi) \neq 0$.

b). This forms $\Gamma(z) = 0$ or $\Gamma(1-z) = 0$ do not have roots in C-Z.

c). Solution of $\sin(\pi/2z) = 0$ or $\cos(\pi/2z) = 0$. More specifically if $z = x + yi$ then...

$$1. \sin(\pi/2z) = 0 \Rightarrow \cosh[(\pi \cdot y)/2] \sin[(\pi \cdot x)/2] + i \cdot \cos[(\pi \cdot x)/2] \sinh[(\pi \cdot y)/2] = 0$$

Has a solution

If $k, m \in \mathbb{N}(\text{Integers})$ then the solution is

$$i) x = \frac{2(-\frac{\pi}{2} + 2 \cdot \pi \cdot m)}{\pi} \text{ or } x = \frac{2(\frac{\pi}{2} + 2 \cdot \pi \cdot m)}{\pi} \text{ and } y = i(-1 + 4 \cdot k) \text{ or } y = i(1 + 4 \cdot k) \text{ and}$$

$$ii) x = \frac{2(\pi + 2 \cdot \pi \cdot m)}{\pi} \text{ or } x = 4 \cdot m \text{ and } y = i \cdot (4 \cdot k) \text{ or } y = 2 \cdot i \cdot (1 + 2 \cdot k) \text{ and}$$

with the result of the program mathematica

Of the generalized solution it seems that the pairs (x, y) will always arise integers which make impossible the case is $0 < x < 1$, therefore there are no roots of the equation **$\sin(\pi/2z) = 0$ because as we see all the roots it is $x > 1$.**

$$2. \cos(\pi/2z) = 0 \Rightarrow \cosh[(\pi y)/2] \sin[(\pi x)/2] + i \cos[(\pi \cdot x)/2] \sinh[(\pi y)/2] = 0$$

It has a solution If $k, m \in \mathbb{N}(\text{Integers})$ and the solution is

$$i) x = \frac{2(-\frac{\pi}{2} + 2 \cdot \pi \cdot m)}{\pi} \text{ or } x = \frac{2(\frac{\pi}{2} + 2 \cdot \pi \cdot m)}{\pi} \text{ and } y = i(4 \cdot k) \text{ or } y = 2 \cdot i(1 + 2 \cdot k) \text{ and}$$

$$ii) x = \frac{2(\pi + 2 \cdot \pi \cdot m)}{\pi} \text{ or } x = 4 \cdot m \text{ and } y = i \cdot (-1 + 4 \cdot k) \text{ or } y = i \cdot (1 + 4 \cdot k) \text{ and}$$

with the result of the program mathematica.

As we can see, again of the generalized solution it seems that the pairs (x, y) will always arise integers which make impossible the case is $0 < x < 1$, therefore there are not roots of the equation **$\cos(\pi/2z) = 0$.**

With these three cases, we exclude the case one of these two equations(b,c) be equated to zero for $z = x + yi$ and we have validated concurrently $0 < x < 1$, because x from that shown in \mathbb{Z} .

II) Therefore we analyze the two equations of the Riemann zeta function and try to find common solutions.

1. For the first equation and for real values with $\operatorname{Re} s > 0$ apply:

$$\zeta(1-s)/\zeta(s) = 2(2\pi)^{-s} \cos(\pi \cdot s/2) \Gamma(s) \Rightarrow \zeta(1-s) = f(s) \cdot \zeta(s), \text{ where}$$

$$f(s) = 2(2\pi)^{-s} \cos(\pi s/2) \Gamma(s) \text{ but this means that the following two cases occur:}$$

$$a. \zeta(1-s) = \zeta(s), s \text{ complex number.}$$

This assumption implies that $\zeta(s)(1-f(s)) = 0 \Rightarrow \zeta(s) = 0$. In #3 (page 1), we showed that the function $\zeta(s)$ is 1-1 and if $\zeta(x_0 + y_0 i) = \zeta(x'_0 + y'_0 i)$ then $y_0 = y'_0 \wedge x_0 = x'_0$, but we also apply that $\zeta(x_0 + y_0 i) = \zeta(x_0 - y_0 i) = 0$, because apply for complex roots. The form

$\zeta(1-s) = \zeta(s)$ means if $s = x_0 + y_0i$ that apply $1 - x_0 = x_0$ namely $x_0 = 1/2$ because in this case it will be verified $\zeta(1 - x_0 - y_0i) = \zeta(1/2 - y_0i) = \zeta(1/2 + y_0i) = 0$ and like that it can be verified the definition of any complex equation. Therefore if $s = x_0 + y_0i$ then $x_0 = 1/2$ to verify the equation $\zeta(s) = 0$.

b. $\zeta(1-s) \neq \zeta(s)$, when s is a complex number.

The case for this to be verified should be $\zeta(s) = \zeta(1-s)f(s) \Rightarrow \zeta(s) = 0 \wedge f(s) = 0$. But this case is not possible, because as we have shown in **Section i(b)**, the individual functions of $f(s)$ cannot be zero when s is a complex number.

2. For the second equation and for any real values with $\text{Re } s < 1$ we apply

$\zeta(s)/\zeta(1-s) = 2(2\pi)^{s-1} \text{Sin}(\pi \cdot s/2)\Gamma(1-s) \Rightarrow \zeta(s) = f(s) \cdot \zeta(1-s)$, where
 $f(s) = 2(2\pi)^{s-1} \text{Sin}(\pi \cdot s/2)\Gamma(1-s)$ but this also means that two cases occur:

a. $\zeta(1-s) = \zeta(s)$, when s is a complex number.

This case is equivalent to 1.a, and therefore if $s = x_0 + y_0i$ then $x_0 = 1/2$ in order to verify the equation $\zeta(s) = 0$.

b. $\zeta(1-s) \neq \zeta(s)$, where s is a complex number.

Similarly, the above case is equivalent to 1.b and therefore it cannot be happening, as it has been proved.

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Blow up of solutions for a nonlinear system of fractional differential equations

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Abstract

The aim of this work is to enquire into the blow-up in finite time of solutions for the following Cauchy value problem of fractional ordinary differential equations

$$\begin{aligned}u_t + a {}^c D^\alpha(u) &= \frac{1}{\Gamma(1-\gamma_1)} \int_0^t (t-s)^{-\gamma_1} |v(s)|^{q-1} v(s) ds \quad t > 0, \\v_t + b {}^c D^\beta(v) &= \frac{1}{\Gamma(1-\gamma_2)} \int_0^t (t-s)^{-\gamma_2} |u(s)|^{p-1} u(s) ds \quad t > 0,\end{aligned}$$

where Γ is the Euler Gamma function and $p > 1$, $q > 1$, $a, b > 0$, are positive constants, $0 < \alpha, \beta < 1$, $0 < \gamma_1, \gamma_2 < 1$.

Further, ${}^c D^\alpha$, ${}^c D^\beta$ denote the Caputo fractional derivatives of order α , β , respectively.

Two main contributions are presented in this work: First, we prove the existence and uniqueness of local classical solutions for the our system supplemented with initial data, then we establish a result on the blow-up in finite time of these solutions.

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ON SOME PROPERTIES OF A WEAKLY I_{rg} -CLOSED SET

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ABSTRACT

In 2013, a generalized class of τ^* called weakly I_{rg} -open sets in ideal topological spaces was introduced and weakly I_{rg} -closed sets in ideal topological spaces were studied by Ekici and Özen [A generalized class of τ^* in ideal spaces, Filomat, 27 (4) (2013), 529-535]. Some new properties of weakly I_{rg} -closed sets are investigated in this paper.

Keywords: weakly I_{rg} -open set, weakly I_{rg} -closed set, τ^* , ideal

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Further revelations result from regarding Hilbert's 'marks on paper' as primitive

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Abstract: This paper continues with the approach from first principles [1] demonstrated at the 2016 conference where some of the conclusions of Russel, Cantor and Gödel regarding the foundations of mathematics were addressed. By starting with Hilbert's 'marks on paper' it was shown that those conclusions were in fact meaningless. How this was done is summarised in this paper and the same methodology is used to show the error in Galileo's "paradox" where the squares of the positive integers are placed in a 1-1 correspondence with the positive integers themselves [2]. Such a bijection is shown to be spurious because the correspondence is not between numbers but between digits and numbers. It requires justification to claim it exists for numbers and it is argued that this is not forthcoming since although the digits of the set of integers are normally regarded as representing numbers because they belong to an arithmetical language, only those defined here by the result of squaring can be regarded as such. It is pointed out that the error is similar to that of not distinguishing between the use and mention of a name which is often emphasised in the philosophy of language. This means that only digits that are used can be regarded as representing numbers. A digit merely mentioned in a list remains a Hilbert 'mark on paper'. It is indicated that without the function f as $f(n) = n^2$ from the set of whole numbers to the subset of squares of the whole numbers then it is not the case that an infinite set has the same 'cardinal' as a subset of itself. The more general consequences for infinite sets of requiring operations on digits to be underwritten by operations on numbers is discussed and it is shown that the Dedekind definition of infinite sets [3] does not hold.

Key words: Foundations of mathematics, infinite sets; paradox.

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Coefficient problem for bounded analytic functions with Hilbert space codomain

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Abstract

We study Taylor coefficient problem for bounded analytic functions acting from the open unit disc of the complex plane to the complex Hilbert space H .

Let H^n be the n -th power of the space H , $H^n = \{ \{A^k\}_0^{n-1}, A_k \text{ belongs to } H \}$, and let $K(H^n)$ be a subset of H^n with all elements $\{A^k\}_0^{n-1}$ that are strings of the first n Taylor coefficients of bounded analytic functions acting from the open unit disc of the complex plane to H (coefficient body).

It has been shown in the present paper, that $K(H^n)$ is a closed set in the space H^n , and description of the boundary points of $K(H^n)$ has been found in terms of solution to the corresponding problem for functions with finite dimensional Euclidean space codomain.

Key words: Hilbert space, analytic function, coefficient problem

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Mobility of Objects in Space: Some Recent Results and Open Problems

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Abstract

This paper describes some recent results concerned with the mobility of objects in a R^2 and R^3 . One class of problems asks simply whether two or more objects can be separated by one or more translations or rotations, without any collisions occurring between the objects during the motions. Such objects may consist of discs and balls [6], [7], a variety of different classes of simple polygons [10], [15], and polyhedra [1], [4], [8], [11]. Another class of problems is defined for polygonal and polyhedral linkages in which their vertices and edges, respectively, act as joints (either revolute, universal, or hinge). Such problems ask whether the linkages can be reconfigured from one configuration to another, by means of specified allowed joint motions [2], [3], [5], [9], [12], [13], [14]. Several open problems in this area are indicated.

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