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Estimation of the System Reliability under Type-II Right Censored Ranked Set Sampling Data

Fatma Gül AKGÜL, Birdal ŞENOĞLU

*Department of Statistics, Faculty of Science, Ankara University, ANKARA, TURKEY
E-mail: fgakgul@ankara.edu.tr, senoglu@ankara.edu.tr*

Estimation of the system reliability is very important especially in engineering studies. Therefore, there have been various attempts to solve this problem in the statistics literature. In the estimation procedure either we have complete samples or we have censored samples. In our earlier studies, we obtained different estimators of the system reliability $R=P(X<Y)$ under the simple random sampling (SRS), the ranked set sampling (RSS) and the modifications of RSS, see Akgül and Şenoğlu [1,2] and Akgül [3]. In these studies, we assumed that the samples corresponding to the stress X and the strength Y are complete. Finally, we compared their efficiencies by using an extensive Monte Carlo simulation study.

In the present study, we extended our previous studies to the censored samples case. We assumed that the samples for both X and Y are type-II right censored Weibull. Simulation results showed that estimators based on RSS are more efficient than the corresponding estimators based on SRS, see Akgül [3].

Keywords: Stress-strength model, Ranked set sampling, Type-II right censoring, Weibull, Monte-Carlo simulation.

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Calculation of the temperature distribution field in the deformation zone in metal rod rolling with the aid of locally homogeneous scheme

Ospanova T. T., Sharipbayev A.A., Niyazova R.S.

L.N.Gumilyev Eurasian National University, Astana city, Kazakhstan

On the basis of heat-conduction equation is solved the three-dimensional problem of temperature distribution field for continuous casting and rolling line of metal rod. Wherein considered the temperature rate change in rolling mill and intercellular gap of rolling mill line.

In order to solve three dimensional problem of heat-conduction is used the score approximation method of locally one-dimensional schemes. With the aid of which multivariate problem reduced to locally one-dimensional problem. Calculation of temperature field distribution in deformation zone is executed under inherent scheme.

Keywords: continuous casting, a metal rod, distribution field, deformation zone, stand, locally one-dimensional scheme.

Derivations on proper CQ^* -algebras

Choonkil Park

Department of Mathematics, Hanyang University, Republic of Korea

E-Mail: baak@hanyang.ac.kr

We investigate homomorphisms in proper CQ^* -algebras and proper Lie CQ^* -algebras, and derivations on proper CQ^* -algebras and proper Lie CQ^* -algebras associated with the Cauchy-Jensen functional equation

$$2f\left(\frac{x+y}{2} + z\right) = f(x) + f(y) + 2f(z).$$

Isometries and isometric isomorphisms in proper CQ^* -algebras are studied.

2010 MSC. Primary 47N50, 47L60, 39B52, 39B72, 47L90, 46H35, 46B03, 47Jxx,
17A40, 17C65, 46K70, 47B48, 46L05.

New Equations for Involute-Evolute Curves in Euclidean 3-Space

Vildan Özdemir (Bacak)*

Department of Mathematics
Selçuk University, Konya, Turkey

Nihat Ayyıldız†

Department of Mathematics
Süleyman Demirel University, Isparta, Turkey

Abstract

In this study, we survey space Euclidean curves with positive curvature κ , torsion τ and spherical normal parametrization σ such that the radius of curvature κ^{-1} satisfies an eigenvalue equation. We characterize the equations in terms of evolutes and involutes of a space curve c and give relations between evolutes and involutes. We get inhomogeneous eigenvalue equations. Also, we give some equations for support function.

Keywords. Curvature function, eigenvalue equations, Euclidean space curves, evolutes, involutes, support function

*email: vildanbacak@selcuk.edu.tr; *Corresponding author

†email: nihatayyildiz@sdu.edu.tr

Solution of Lane-Emden Type Equations Using Polynomial-Sinc Collocation Method

M. Youssef^{*1} and G. Baumann^{†1,2}

¹Mathematics Department, Faculty of Basic Science, German University in Cairo,
New Cairo City 11835, Egypt

²University of Ulm, Albert-Einstein-Allee 11, D-89069, Ulm, Germany

Abstract

The Lane-Emden (LE) equation is one of the basic equations in the theory of stellar structure and has been the focus of several studies [1,2,3]. In general, LE type equations are nonlinear ODEs that can be formulated as:

$$y'' + \frac{2}{x}y' + g(y) = R(x), \quad x > 0 \quad (1)$$

with the initial conditions,

$$y(0) = a, \quad y'(0) = b.$$

In general the generic function $g(y)$ is nonlinear and $R(x)$ is a function of x only. Many problems in mathematical physics and astrophysics are related to this equation. Some of these applications are homogeneous, $R(x) = 0$, and others are inhomogeneous. For every physical application a suitable choice of the generic function $g(y)$ is made.

One of the main problems with the LE equation is the singularity at $x = 0$ which is a singularity at the boundary as well as of the equation. This singularity was a challenge for many scholars to numerically represent the solution of the LE equation.

In this paper we introduce a series solution based on Lagrange polynomials that can easily deal with singularity problems. The proposed approximation is based on non-equidistant interpolation points generated by conformal maps. Our method provides the solution by an exponential convergent series. This exponential convergence property arises from the use of Sinc points as interpolation points in the Lagrange polynomials. We examine the technique for different types of Emden' equations and compare the solution with Taylor approximation. In addition the error formula shows exponential convergence.

*maha.youssef@guc.edu.eg

†gerd.baumann@guc.edu.eg

A Column Generation for Ship Routing and Scheduling Problem with Time Window

Khaled Al-Hamad

College of Technological Studies, Public Authority for Applied Education and Training, Kuwait.

Email: km.alhamad@paaet.edu.kw

Abstract–

This paper develops an efficient optimal approach of a Column Generation for a ship routing and scheduling problem (SRSP) with time-window in industrial shipping operation mode. This method addresses the problem of loading shipments for many customers using heterogeneous ships (controlled and chartered ships, where the operations manager can resort to the market for spot chartered ships at a given cost). There are a number of cargos to be shipped. Each cargo consists of a given quantity to be loaded at the origin and delivered to a cargo port. For any cargo there are time-window constraints on the earliest and latest time for arrival at each cargo port. No ship is allowed to arrive outside the time-window. Constraints relate to delivery time windows imposed by customers, the time horizon by which all deliveries must be made and ship capacities.

The Column Generations adopted to solve this type of problem is based on the Set Partitioning Problem (SPP). Column Generations uses a procedure that generates a set of feasible candidate schedules, each of which corresponds to a variable in the SPP model. Each individual candidate schedule has a route for a specific ship containing one or more shipments. A number of candidate schedules for a specific ship are represented by a subset. The union for all subsets (for the set of all ships in the fleet) forms a set of candidate schedules.

Since this type of problem considered as NP-hard, it is found that while the optimal approach solves small scale problem efficiently, treating large scale problems with the exact method becomes involved due to computational problem. The goal of this research is to solve large scale problem in efficient way. This can be accomplished by finding an approach capable to minimise the number of feasible candidate schedules by selecting more promising candidate schedules.

OPTIMAL CONTROL PROBLEMS OF STOCHASTIC FLOWS WITH RUNGE-KUTTA SCHEMES

Fikriye Nuray Yılmaz

Gazi University, Ankara, Turkey

email:yfikriye@gmail.com

Recently, optimal control of stochastic partial differential equations (SPDEs) have been an active research field in mathematics and physics. A lot of work has been done on control of stochastic differential equations (SDEs). However, there are not many works on the stochastic control of SPDEs. Similar to deterministic optimal control problems, numerical methods have been proposed for stochastic problems. In this work, we consider the optimal control problems of SPDEs with additive white noise. There are classical approaches for control problem to derive necessary optimality conditions. Standard optimization algorithms require the gradient computations of the cost functional given. In this work, we use discretize-then-optimize approach to get the first order optimality conditions. We use a deterministic discretization scheme for space variable. We apply finite element method to get semi-discrete state equation. We focus on time discretization with stochastic Runge-Kutta method. After we get the optimality system, we use a gradient descent algorithm for implementing the optimization problem. Finally, we perform some numerical examples.

On a filtered probability space, $(\Omega, \mathcal{F}, \mathbb{P})$ with $\mathcal{F} = (\mathcal{F}_t)_{t \geq 0}$, we are interested in minimizing the expected cost function

$$J(y, u) := \mathbb{E} \left[\int_0^T g(s, y_s, u_s) ds + f(y_T) \right],$$

where f and g are integrable functions. u is the pointwise control variable. Our control problem is

$$\underset{u}{\text{minimize}} \quad J(y, u)$$

subject to

$$\begin{cases} dy = (\varepsilon \Delta y - b \cdot \nabla y - ry + u)dt + \gamma dW \\ y(0, \cdot) = y_0, \end{cases}$$

where $\varepsilon = \frac{1}{Re}$ denotes the viscosity parameter and Re is the Reynolds number, b is the fluid velocity, r is a reaction coefficient. And γ is a real-valued function. W is the standard Brownian motion. y_0 is the initial condition.

We use Monte-Carlo simulation method in order to estimate conditional expectations. Monte-Carlo method is robust and it generates sample of investigated states from random realizations of the input data.

We compare the numerical results obtained from Runge-Kutta scheme and Euler approximation.

Inhomogeneous Plane Waves in Cubic Crystals subject to a Bias

LIVIA HARABAGIU

Department of Mathematics
University of Bucharest
Bucharest, Romania
harabagiu_livia@yahoo.com

Abstract

In the present paper we investigate the condition of inhomogeneous plane waves propagation in cubic crystals subject to initial deformations and electric fields. The author obtains here the components of the electroacoustic tensor and the velocities of propagation as closed-form solutions. We show the influence of electrostrictive and piezoelectric effects on wave propagation in such media. We analyze the influence of the initial fields on the waves polarization in two main cases: (i) propagation in isotropic directional bivectors; (ii) propagation in case of polar anisotropic directional bivectors.

Keywords—Inhomogeneous plane waves, cubic crystals, initial fields, isotropic/anisotropic directional bivectors.

Mathematics Subject Classification(2010): 74J05, 74B15, 74E15, 74F15, 51P05, 74N05.

TOPOLOGICAL SOFT SUBSTRUCTURES OF TOPOLOGICAL GROUPS AND TOPOLOGICAL RINGS

H. FULYA AKIZ* AND AKIN O. ATAGUN

ABSTRACT. A *topological group* is a group G together with a topology on G such that the group's binary operation and the group's inverse function are continuous functions with respect to the topology([1],[2]). Further, a *topological ring* is a ring R which is also a topological space such that both the addition and the multiplication are continuous as maps $R \times R \rightarrow R$, where $R \times R$ carries the product topology ([2],[3],[4]). Every topological ring is a topological group.

Soft set theory was proposed by Molodtsov [5] in 1999. A soft set, over a universal set X and set of parameters E is a pair (F, A) where A is a subset of E , and F is a function from A to the power set of X . In other words, this defines a parameterised family of subsets of X . For each $e \in A$, the set $F(e)$ is called the *value set* of e in (F, A) . At present, work on soft set theory is progressing rapidly. Some operations and application of soft sets were studied by many researchers.

In [6], soft subrings and soft ideals of a ring are introduced and studied by using Molodtsov's definition of the soft sets. In this paper, we study on the topological version of [6]. Firstly, we introduce and study topological soft subgroups of a topological group. Moreover, we introduce topological soft subrings of a topological rings. Some related properties about topological soft substructures of topological rings are investigated and illustrated by many examples.

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1 DEPARTMENT OF MATHEMATICS, BOZOK UNIVERSITY
E-mail address: hfulya@gmail.com

2 DEPARTMENT OF MATHEMATICS, BOZOK UNIVERSITY
E-mail address: aosman.atagun@bozok.edu.tr

Key words and phrases. Topological rings, soft rings, topological soft rings.

SOME GENERALIZATIONS OF SOFT MATRIX PRODUCTS

HÜSEYİN KAMACI, AKIN OSMAN ATAGÜN* AND HÜRMET FULYA AKIZ

ABSTRACT. Soft set theory, introduced by Molodtsov [4], has been regarded as an impressive mathematical tool to deal with uncertainties. Soft matrices, defined in [1], are more functional to make theoretical studies in the soft set theory. Soft matrix representation has several advantages, for example since it is easy to store and manipulate matrices, then we can represent a corresponding soft set in a computer. Çağman and Enginoğlu [1] defined \wedge -product, \vee -product, $\bar{\wedge}$ -product and $\bar{\vee}$ -product of soft matrices. In [1], by the definition of \wedge -product, \vee -product, $\bar{\wedge}$ -product and $\bar{\vee}$ -product, we see that the soft matrices have to be same type for the definitions are meaningful. In this paper, we define some new products of different type of soft matrices by generalizing the products of soft matrices given in [1], and study their algebraic properties, such as associativity, commutativity, identity elements, De Morgan's laws, etc.

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ADDRESS1 DEPARTMENT OF MATHEMATICS, BOZOK UNIVERSITY
E-mail address: huseyin.kamaci@bozok.edu.tr

ADDRESS2 DEPARTMENT OF MATHEMATICS, BOZOK UNIVERSITY
E-mail address: aosman.atagun@bozok.edu.tr

ADDRESS3 DEPARTMENT OF MATHEMATICS, BOZOK UNIVERSITY
E-mail address: fulya.gencel@bozok.edu.tr

Key words and phrases. soft sets, soft matrix, products of soft matrices.

An Algorithm for Explicit Form of Fundamental Units of Certain Real Quadratic Fields

Özen ÖZER

Faculty of Science and Arts, Department of Mathematics, Kırklareli University ,

KIRKLARELI-TURKEY

e-mail: ozenozer@klu.edu.tr

Ayten Pekin

Faculty of Sciences, Department of Mathematics, İstanbul University,

İSTANBUL-TURKEY

e-mail: aypekin@istanbul.edu.tr

Abstract: Quadratic fields have applications in different areas of mathematics such as quadratic forms, algebraic geometry, diophantine equations, algebraic number theory, and even cryptography. The Unit Theorem for real quadratic fields says that every unit in the integer ring of a quadratic field is given in terms of the fundamental unit of the quadratic field. Thus determining the fundamental units of quadratic fields is of great importance. In this paper, we obtained an explicit formulation to determine the form of the fundamental units of certain real quadratic number fields. This new algorithm for such quadratic fields is first in the literature and it gives us a more practical way to calculate the fundamental unit. Where, the period in the continued fraction expansion of the quadratic irrational number of the certain real quadratic fields is equal to 7.

1

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THE NUMBERS CONSECRATED IN THE BLIEFS AND RITUALS OF MESOPOTAMIA AND CENTRAL ASIA, HITTITE

Metin Akız*

Aslı Kahraman Çınar

From the ancient time to modern world, numbers have been thought significant within a great deal of religions and enshrined in the rituals. Any numbers have been hallowed by not only divine religions but also non divine religions and people have made a point of praying the days corresponding the numbers. One of these shrined numbers is '7'. By the ritual texts written by ancient people, the number 7 has had importance for the bloody or non bloody victims consecrated to god, dream interpretations and it is thought that the prays considering the number '7' have been more valuable than the others.

In this paper, we study on the religions, prays, vows, victims, mythos, adages, idioms, fables, lullabies and superstitions according to number '7', of Hittites, civilizations of Mesopotamia, Central Asia Societies which are in the Anatolia geography from the ancient time to the present. By getting out of original texts from cuneiform scripts, steppe mythos, written document from Mesopotamia, we compare with some similar still-continuing beliefs according to the number '7'. Further, in this study we give some results and evaluate the reports, obtained from meetings, with people who are living in the different places in Anatolia, in the light of texts from cuneiform scripts. This paper is prepared by the fieldwork which is done in the different places of Anatolia.

Keywords: numbers, relation of numbers, consecrated numbers, numbers in the beliefs and rituals

* Phd Metin Akız, Bozok University Science and Social Faculty, History Department, metinakiz@gmail.com

Research Assistant Aslı Kahraman Çınar, Bozok University, Science and Social Faculty, History Department, aslikahramancinar@gmail.com

Magic Squares from the Law of the Lever

¹Liuhan Oliveira de MIRANDA, ²Lohans de Oliveira MIRANDA*, ³Lossian Barbosa
Bacelar MIRANDA

¹Coordination of Mechanical Engineering, UFPI, lohansmiranda@gmail.com

²Health Sciences Center, UFPI, liuhan.oliveira@gmail.com

³Mathematics Coordination, IFPI, lossian@oi.com.br
Teresina (PI), Brazil

Abstract— We consider a lever with fulcrum at the origin of a one-dimensional linear system of coordinates, with extremities equidistant of the origin. Our initial problem is to establish a one-to-one correspondence between a finite set of point masses and a set of geometric points of the lever so that it stands in rotational equilibrium when the point masses are put on the geometrical points. We assume that the extremities of the lever belong to this set of geometric points; the amount of these geometric points is a multiple of four or successor of multiple of four; the abscissas of these points form a nonconstant arithmetic progression and the masses of point masses, too, form a nonconstant arithmetic progression. When the amount of these geometric points is an odd number the problem reduces to the pair case, since, in this situation, one of the point masses coincide with the fulcrum. Also, due to proportionality, the solution of this initial problem can be achieved from the application of combinatorial techniques and resolution of a homogeneous linear Diophantine equation imposed by the equilibrium condition of the lever. Using constructive techniques of elementary arithmetic, we established a method that provides explicit and general solution to the problem, whatever the amount of point masses. The second, and main problem discussed here, concerns the use of aforementioned method to construct magic squares whose orders are equal to the number of point masses. We solve this problem by establishing a second method that constructs square matrices whose rows and columns are solutions of the above mentioned Diophantine equation. In addition, the sums of the elements of the diagonals of these matrices are equal to the magic constant. If we change these stable matrices, doing swaps of elements of different lines along of columns and swaps of elements of different columns along of lines, we obtain a broad class of magic squares. The rows and columns of these magic squares are solutions of the Diophantine equation.

Keywords— Law of the lever; magic squares; Diophantine equations; permutations.

An integral formula for a finite sum of inverse powers of cosines

Carlos M. da Fonseca, Department of Mathematics, Kuwait University, Kuwait

M. Lawrence Glasser, Department of Physics, Clarkson University, Potsdam, NY, USA

Victor Kowalenko, Department of Mathematics and Statistics, The University of Melbourne, Victoria, Australia

Abstract

We present a new integral formula for the Gardner -Fisher trigonometric power sum

$$S_{m,v} = \left(\frac{\pi}{2m}\right)^{2v} \sum_{k=1}^{m-1} \cos^{-2v}\left(\frac{k\pi}{2m}\right),$$

where m and v are positive integers.

Modelling Effective Slip Law at thin Porous Interface via two scale Homogenization techniques

Manoj K. Yadav^{*}, Jan Zeman

Department of Mechanics, Faculty of Civil Engineering, Czech Technical University in Prague

Abstract

We study the problem of obtaining effective slip interface law at a thin rigid porous interface. The underlying flow is a two-phase steady incompressible flow governed by Navier-Stokes equation. Due to the presence of thin porous interface, boundary layers are encountered near the interface. The no-slip condition breaks down in the course of homogenization. To obtain the effective slip law at the interface in the homogenized limit, we upscale the flow in a thin neighbourhood of the rigid porous interface. We apply two scale homogenization techniques to the up-scaled flow under appropriate boundary conditions to obtain the effective slip interface law. The appropriate boundary conditions turn out to be a mixture of both Dirichlet and Neumann types. This approach is in marked difference with the perturbation and matching asymptotic expansion approach. Here the asymptotic expansion is developed from perturbation to the zero order solution obtained by imposing no-slip boundary condition at the interface.

Multiscale analysis of the acoustic diffraction by a large number of small inhomogeneities and applications to metamaterials and cloaking

Mourad Sini

Radon Institute, Austrian Academy of Sciences

and

Department of Mathematics, Kuwait University

Let M be the number of bounded and regular bodies $D_j, j=1, \dots, M$, having a maximum radius $a, a \ll 1$, located in a bounded domain of \mathbf{R}^3 . We are concerned with the time harmonic acoustic scattering problem by a very large number of such small bodies, namely as $M:=M(a):=O(a^{-s}), s > 0$, as a tends to zero. These small bodies are arbitrarily distributed with a minimum distance between them of the order $d:=d(a):=O(a^t)$, as a tends to zero, with t in an appropriate range.

We derive the limiting scattering problems when a goes to zero and provide explicit error estimates in terms of a . We need no periodicity (and no randomness) in distributing such small bodies. We show that there exists a threshold $s^*, s^* > 0$, such that the nature of limiting problem changes if $s < s^*$ or $s = s^*$ or $s > s^*$. In addition, we discuss two applications related to the acoustic metamaterials and cloaking. Precisely, we show how a perforation, of a material modeled by a given refraction index, with small holes of impedance type allows us

- to change the sign of the refraction index from positive to negative values and hence *to design acoustic metamaterials*.
- to modify the refraction index to make it identical to the one of the background medium and hence *to cloak it*.

Fourier-Hilbert Transform for Certain Space of Generalized Functions

S. K. Q. Al-OMARI

Abstract–

In this paper, we give a generalization of the Fourier-Hilbert transform on a class of Boehmians. Further, we show that the Fourier- Hilbert transform of a distribution is distribution which is analytic in the space of distributions of compact support. Further properties are also obtained.

Keywords– Hilbert Transform; Fourier Transform; Fourier-Hilbert Transform; Distribution Space; Boehmian Space.

On the Convergence of semigroups of mappings in Metric Spaces

Buthinah A. Bin Dehaish

Department of Mathematics
Faculty of Science For Girls, King Abdulaziz University
Jeddah 21593, Saudi Arabia.
E-mail:bbendehaish@kau.edu.sa

Abstract

In this talk, we discuss the behavior of the fixed points set of nonexpansive semigroups of nonlinear mappings $\{T_t\}_{t \geq 0}$ i.e. a family such that $T_0(x) = x$, $T_{s+t} = T_s(T_t(x))$, where the domain is a metric space $(M; d)$. In this context we use the generalization of Mann iteration process to study such approximation of the fixed points.

Differential Sandwich Theorems for p -valent Analytic Functions Defined by Cho–Kwon–Srivastava Operator

Badriah A.S.Alamri
 Department of Mathematics
 Faculty of Science
 King Abdulaziz University, Jeddah, Saudi Arabia
baalamri@kau.edu.sa

Abstract. By making use of Cho–Kwon–Srivastava operator, we obtain some subordinations and superordinations results for certain normalized p -valent analytic functions. Some of our results improve and generalize previously known results.

1. Introduction.

Let $H(U)$ be the class of analytic functions in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$, and let $H[a; p]$ be the subclass of $H(U)$ consisting of functions of the form :

$$f(z) = a + a_p z^p + a_{p+1} z^{p+1} + \dots \quad (a \in \mathbb{C}),$$

For simplicity, $H[a] = H[a; 1]$. Let $A(p)$ denote the class of functions $f(z)$ of the form:

$$(1.1) \quad f(z) = z^p + \sum_{n=1}^{\infty} a_n z^{n+p}. \quad (p \in \mathbb{N} = \{1, 2, \dots\}),$$

Which are analytic and p -valent in U .

If $f, g \in H(U)$, we say that the function f is subordinate to g , or the function g is superordinate to f , if there exists a Schwarz function ω , i.e., $\omega \in H(U)$ with $\omega(0) = 0$ and $|\omega(z)| < 1, z \in U$, such that $f(z) = g(\omega(z))$ for all $z \in U$. This subordination is usually denoted by $f(z) \prec_p g(z)$. It is well known that, if the function g is univalent in U , then $f(z) \prec_p g(z)$ is equivalent to $f(0) = g(0)$ and $f(U) \subset g(U)$.

Supposing that p, h are two analytic functions in U , let

$$\varphi(r, s, t; z) : \mathbb{C}^3 \times U \rightarrow \mathbb{C}.$$

If $p(z)$ and $\varphi(p(z), zp'(z), z^2 p''(z); z)$ are univalent functions in U and $p(z)$ satisfies the second-order differential subordination

$$(1.2) \quad h(z) \prec \varphi(p(z), zp'(z), z^2 p''(z); z),$$

then $p(z)$ is called to be a solution of the differential superordination (1.2). Analytic function $q(z)$ is called a subordinant of the solution of the differential superordination (1.2), if $q(z) \prec p(z)$ for all the functions $p(z)$ satisfying (1.2). A

univalent subordinate ϑ that satisfies $q(z) \prec \vartheta(z)$ for all of the subordinants q of (1.2), is called the best subordinate.

Recently, Miller and Mocanu obtained sufficient conditions on the functions h , q and φ for which the following implication holds:

$$h(z) \prec \varphi(p(z), zp'(z), z^p p''(z); z) \Rightarrow q(z) \prec p(z).$$

For functions $f_j(z) \in A(p)$, given by

$$f_j(z) = z^p + \sum_{n=1}^{\infty} a_{n,j} z^{n+p} \quad (j = 1, 2),$$

we define the Hadamard product (or convolution) of $f_1(z)$ and $f_2(z)$ by

$$(f_1 * f_2)(z) = z^p + \sum_{n=1}^{\infty} a_{n,1} a_{n,2} z^{n+p} = (f_2 * f_1)(z) \quad (z \in U).$$

In terms of the Pochhammer symbol $(\theta)_n$ given by

$$(\theta)_n = \begin{cases} 1, & (n = 0) \\ \theta(\theta+1)\dots(\theta+n-1), & (n \in \mathbb{N} = \{1, 2, \dots\}), \end{cases}$$

we now define a function $\varphi_p(a, c; z)$ by

$$(1.3) \quad \varphi_p(a, c; z) = z^p + \sum_{n=1}^{\infty} \frac{(a)_n}{(c)_n} z^{n+p} \\ (a \in \mathbb{R}; c \in \mathbb{R} \setminus Z_0^-; Z_0^- = \{0, -1, -2, \dots\}; z \in U).$$

With the aid of the function $\varphi_p(a, c; z)$ defined by (1.3), we consider a function $\varphi_p^*(a, c; z)$ given by the following convolution

$$\varphi_p(a, c; z) * \varphi_p^*(a, c; z) = \frac{z^p}{(1-z)^{\lambda+p}} \quad (\lambda > -p; z \in U)$$

which yields the following family of linear operators $I_p^\lambda(a, c)$:

$$(1.4) \quad I_p^\lambda(a, c)f(z) = \varphi_p^*(a, c; z) * f(z) \quad (a, c \in \mathbb{R} \setminus Z_0^-; \lambda > -p; z \in U).$$

For a function $f(z) \in A(p)$, given by (1.1), it is easily seen from (1.4) that

$$(1.5) \quad I_p^\lambda(a, c)f(z) = z^p + \sum_{n=1}^{\infty} \frac{(c)_n (\lambda+p)_n}{(a)_n (1)_n} a_{p+n} z^{p+n} \quad (z \in U),$$

which readily yields the following properties of the operator $I_p^\lambda(a, c)$:

$$(1.6) \quad z \left(I_p^\lambda(a, c) f(z) \right)' = (\lambda + p) I_p^{\lambda+1}(a, c) f(z) - \lambda I_p^\lambda(a, c) f(z)$$

and

$$(1.7) \quad z \left(I_p^\lambda(a+1, c) f(z) \right)' = a I_p^\lambda(a, c) f(z) - (a-p) I_p^\lambda(a+1, c) f(z).$$

The operator $I_p^\lambda(a, c)$ was introduced and studied by Cho et al..

2. Subordination results.

Using Lemma 1, we first prove the following theorem.

Theorem 1. Let $\alpha \neq 0, \beta > 0$ and $q(z)$ be convex univalent in U with $q(0) = 1$. Further assume that

$$(2.1) \quad \operatorname{Re} \left\{ \frac{\beta - p\alpha}{\alpha} + 2q(z) + \left(1 + \frac{zq'(z)}{q(z)} \right) \right\} > 0 \quad (z \in U).$$

If $f, g \in A(p)$ satisfy

$$(2.2) \quad \gamma(f, g, \alpha, \beta) p (\beta - p\alpha) q(z) + \alpha q^2(z) + \alpha z'(z),$$

Where

$$(2.3) \quad \begin{aligned} \gamma(f, g, \alpha, \beta) &= (\beta - (p+1)\alpha) \frac{I_p^{\lambda+1}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} + \alpha \left(\frac{I_p^{\lambda+1}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} \right)^2 \\ &+ \alpha(\lambda + p + 1) \frac{I_p^{\lambda+2}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} \\ &- \alpha(\lambda + p) \frac{I_p^{\lambda+1}(a, c) g(z)}{I_p^\lambda(a, c) g(z)} \left(\frac{I_p^{\lambda+1}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} \right), \end{aligned}$$

then

$$\frac{I_p^{\lambda+1}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} p q(z)$$

And q is the best dominant.

3. Superordination and sandwich results.

Theorem 2. Let $\alpha \neq 0, \beta > 0$. Let q be convex univalent in U with $q(0) = 1$. Assume that

$$(3.1) \quad \operatorname{Re} \{ q(z) \} \geq \operatorname{Re} \left\{ \frac{p\alpha - \beta}{(1+p)\alpha} \right\}.$$

Let $f, g \in A(p)$, $\frac{I_p^{\lambda+1}(a, c) f(z)}{I_p^\lambda(a, c) g(z)} \in H[q(0), 1] \cap Q$, Let $\gamma(f, g, \alpha, \beta)$ be univalent in U

and

$$(3.2) \quad (\beta - p\alpha)q(z) + \alpha q^2(z) + \alpha z q'(z) \prec \gamma(f, g, \alpha, \beta),$$

where $\gamma(f, g, \alpha, \beta)$ is given by (3.3), then

$$(3.3) \quad q(z) \prec \frac{I_p^{\lambda+1}(a, c)f(z)}{I_p^\lambda(a, c)g(z)}.$$

and q is the best subdominant.

We conclude this section by stating the following sandwich result.

Theorem 3. Let q_1 and q_2 be convex univalent in U , $\alpha \neq 0$ and $\beta \geq 1$. Suppose q_2 satisfies (2.1) and q_1 satisfies (3.1). Moreover, suppose

$$\frac{I_p^{\lambda+1}(a, c)f(z)}{I_p^\lambda(a, c)g(z)} \in H[1, 1] \cap Q$$

and $\gamma(f, g, \alpha, \beta)$ is univalent in U . If $f, g \in A(p)$ satisfy

$$(\beta - p\alpha)q_1(z) + \alpha q_1^2(z) + \alpha z q_1'(z) \prec \gamma(f, g, \alpha, \beta) \prec (\beta - p\alpha)q_2(z) + \alpha q_2^2(z) + \alpha z q_2'(z),$$

where $\gamma(f, g, \alpha, \beta)$ is given by (2.3), then

$$q_1(z) \prec \frac{I_p^{\lambda+1}(a, c)f(z)}{I_p^\lambda(a, c)g(z)} \prec q_2(z)$$

and q_1, q_2 are, respectively, the best subdominant and the best dominant.

By taking special values of parameters we get some previously known results.

Multivalued Maps in Modular spaces

Afrah Ahmad Noman Abdou

King Abdulaziz University

Abstract–

The purpose of this paper is to study the existence of fixed points for contractive multivalued maps in the setting of modular metric spaces . The notion of a modular metric on arbitrary set and the corresponding modular spaces, generalizing classical modular over linear spaces like Orlicz spaces, were recently introduced. In this paper we investigate the existence of fixed points of multivalued modular contractive mappings in modular metric spaces. Consequently, our results either generalize or improve fixed point results of Nadler (Pac.J.Math.30:475-488,1969) and Mizaguchi and Takahashi (Journal of Math. Analysis and Application, Vol. 141, no. 1, pp. 177188, 1989).

Asymptotic Behaviour of Resonance Eigenvalues of the Schrödinger Operator with a Matrix Potential

Sedef Erim Karakılıç, Didem Coskan, Setenay Akduman
Department of Mathematics, Dokuz Eylül University,
Buca, Izmir, Turkey

*sedef.irim@deu.edu.tr, coskan.didem@gmail.com, setenayakduman@gmail.com**

Abstract: We study the asymptotic behaviour of the eigenvalues of Schrödinger operator with a matrix potential defined by Neumann boundary condition in $L_2^m(F)$, where F is d –dimensional rectangle and the potential is a $m \times m$ matrix with $m \geq 2$, $d \geq 2$, when the eigenvalues belong to the resonance domain, roughly speaking they lie near planes of diffraction.

Keywords: Schrödinger operator, Neumann condition, Resonance eigenvalue, Perturbation theory.

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